This activity is about colouring, but don't think it's just kid's stuff ☺. This investigation will lead to one of the most famous theorems of mathematics and some very interesting results!!!!

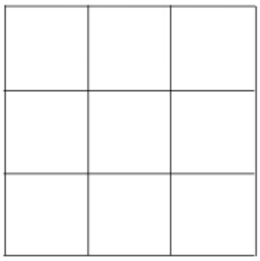
Have you ever coloured in a pattern and wondered **how many colours** you need to use?

There is only one rule

Two sections that share a common edge cannot be coloured the same!

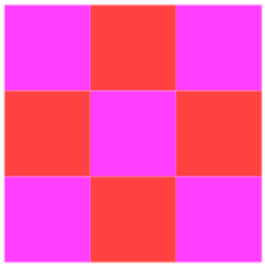
Having a common corner is OK, just not an edge.

Let's start with a simple pattern like a group of nine squares:



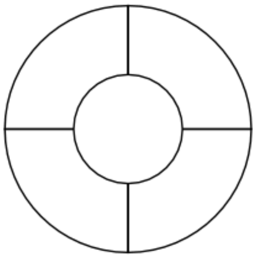
How many colours do you need to colour the pattern of nine squares?

You could use nine different colours, but could make do with as few as **two**:



A Little More Complicated

How about this one?

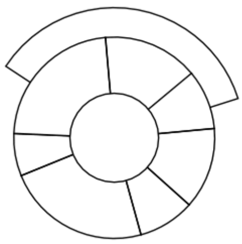


How many colours do you need this time? Try it out...

You couldn't colour this pattern with just two colours. Can you see why?......

Even More Complicated

Let's try another example:



How many colours do you need this time?

Nine? Eight? Seven? Six? Five? Four?

Try it out again

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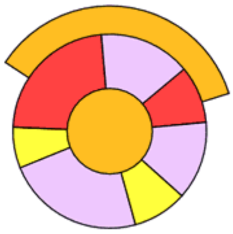
I can't colour this pattern with fewer than four colours



I can change the colours around a bit, but I still need four.

Maps

This could get a bit more interesting if we wanted to colour a map.

A map may not work when a country has two or more separate areas, such as Alaska (part of the US, but with Canada in-between) or Kaliningrad (part of Russia, but also not joined). But let's ignore that here.

Here is a map of part of Europe, showing nine countries and how they border on each other:

Try colouring in the map and see what is the fewest number of colours you need?

Four Colours

It seems that any pattern or map can always be coloured with **four colours**.

In some cases, like the first example, we could use fewer than four. In many cases we could use a lot more colours if we wanted to, but a maximum of **four colours is enough**!

This result has become one of the most famous theorems of mathematics and is known as **The Four Colour Theorem.**

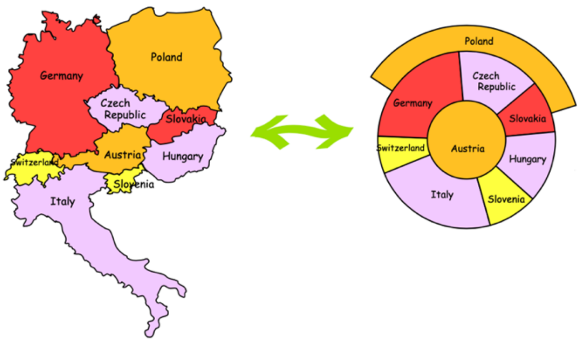
So why is it important?

It is important because it was first stated in 1852, but was not proved until 1976. For over one hundred and twenty years some of the best mathematical brains in the world were unsuccessful in proving one of the simplest theorems in mathematics. There were many false proofs, and a whole new branch of mathematics - known as **Graph Theory (google this)** was developed to try to solve the theorem. But nobody could prove it until in 1976 Appel and Haken proved the theorem with the aid of a computer.

Some people think that, although their proof was correct, it was cheating to use a computer. What do you think?

A Map Can Be Changed!

Now look again at our previous two examples:



Can you see the similarity between these two diagrams?

Imagine the map of European countries was drawn on a piece of rubber that could be stretched. By stretching and skewing the piece of rubber in a certain way, you could end up with the circular diagram.

We say they are **homeomorphic.**

That's a big word, but a very simple idea: **one can become the other**.

It also forms part of a huge branch of mathematics known as **Topology** (again google this).

<u>**One More: US States**</u>

Here is one for you to try on your own ... the "contiguous" (meaning all touching) United States (no Alaska or Hawaii). Can you colour it using only 4 colours?